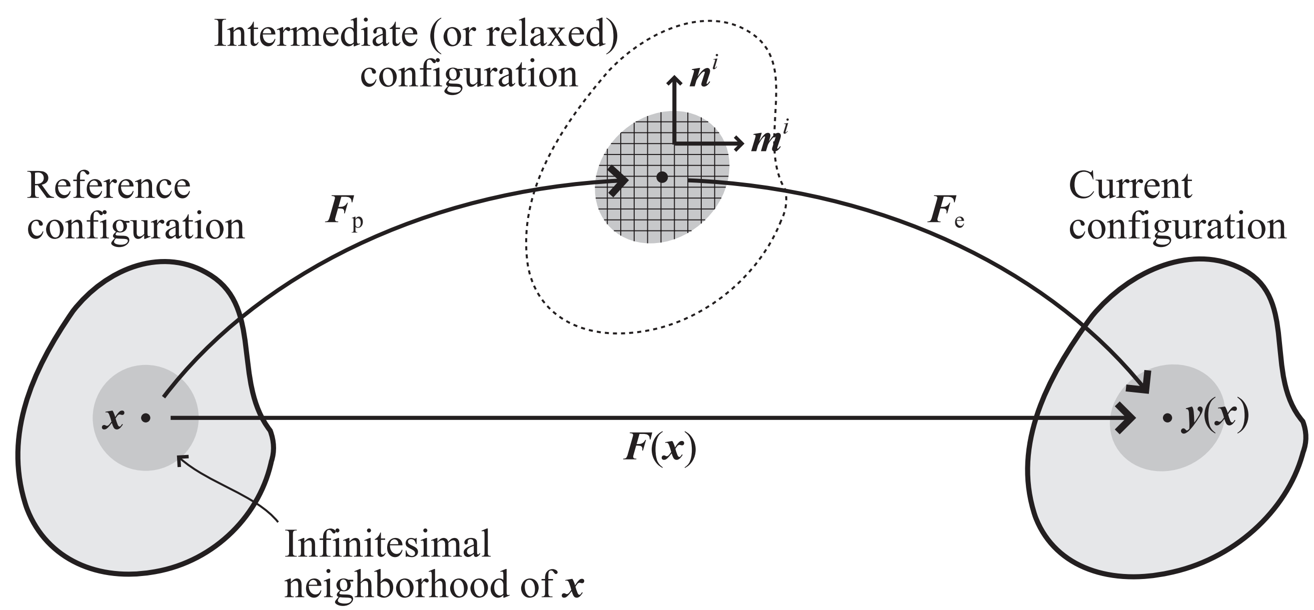
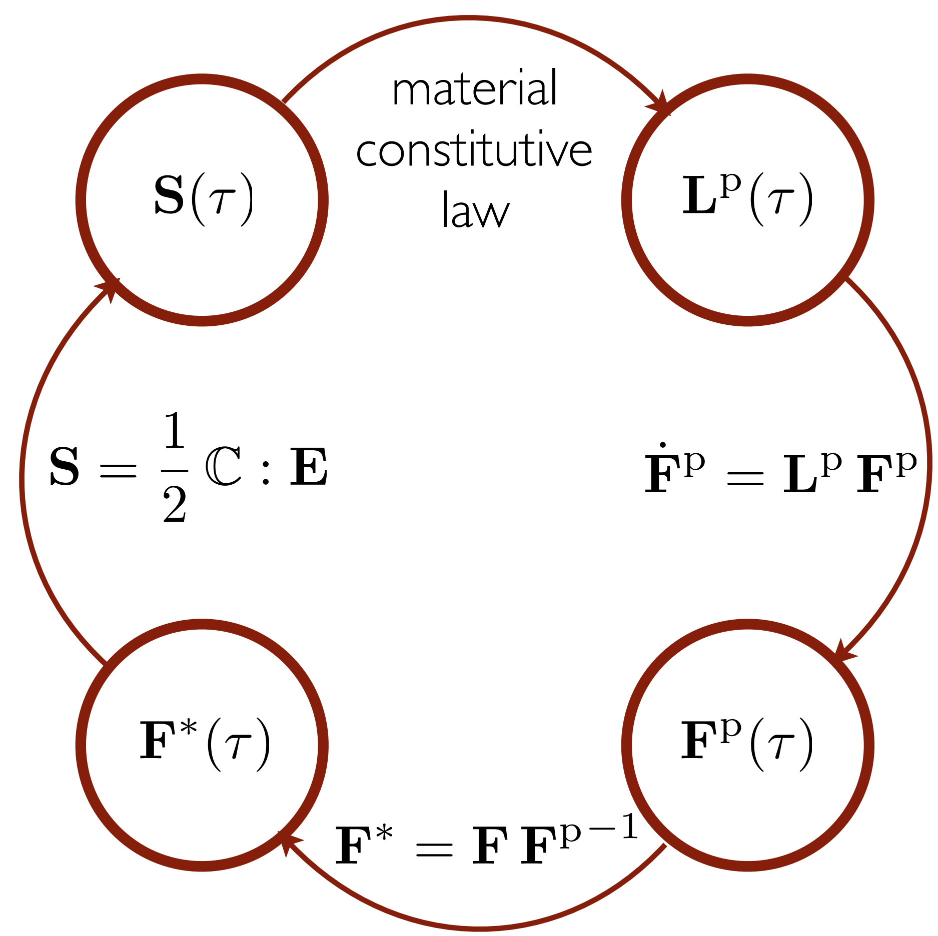
Recollection

Part I & II

- deformation/velocity gradient characterizes *local* (at material point) change/rate of change of shape and size
- material point model connects stress (response) to strain (boundary cond.)
- finite strain plasticity introduces multiplicative decomposition with intermediate configuration
- solution of elastic/plastic strain partitioning requires $\mathbf{L^p}$ from constitutive model





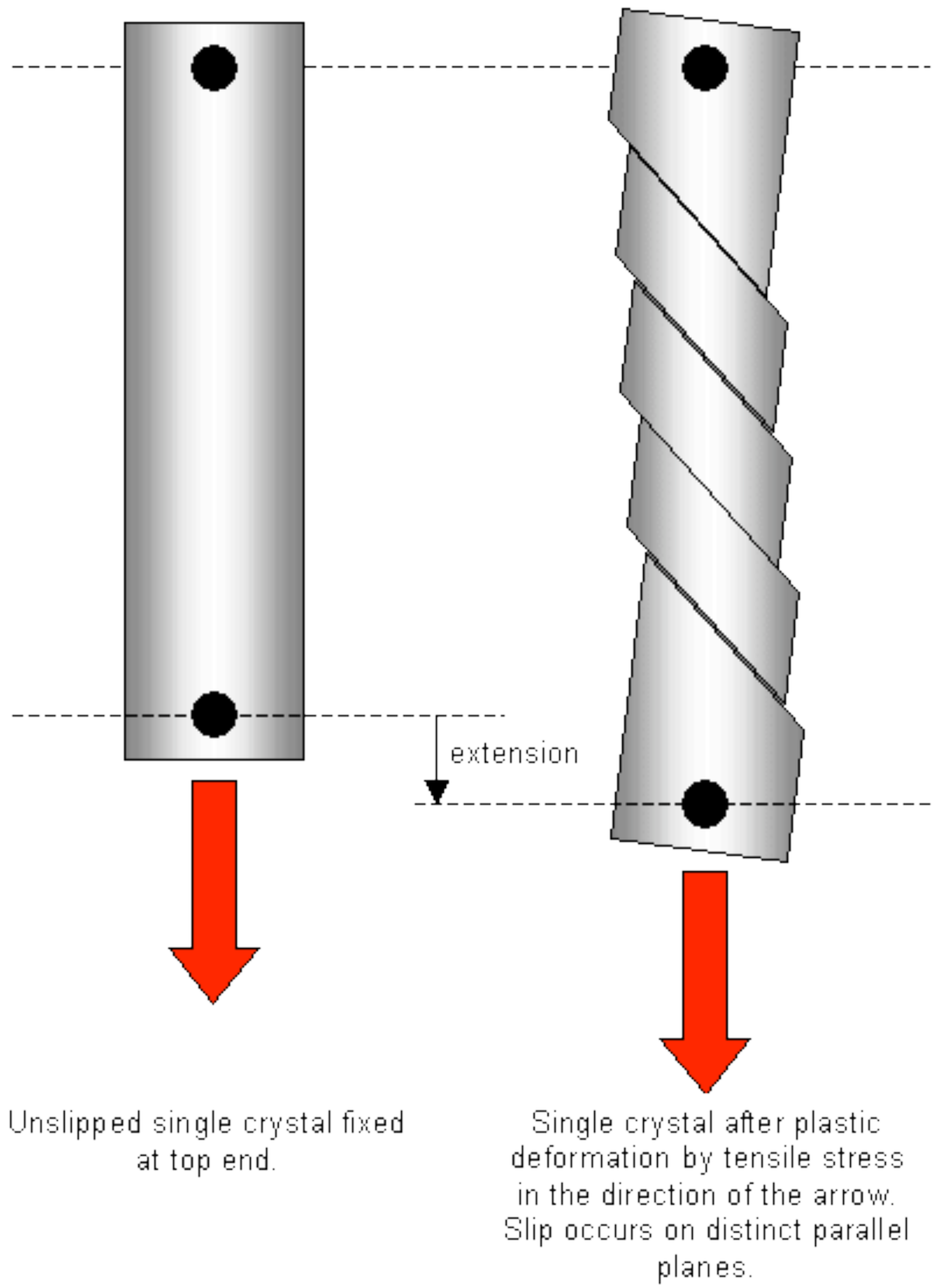
check out the source code...

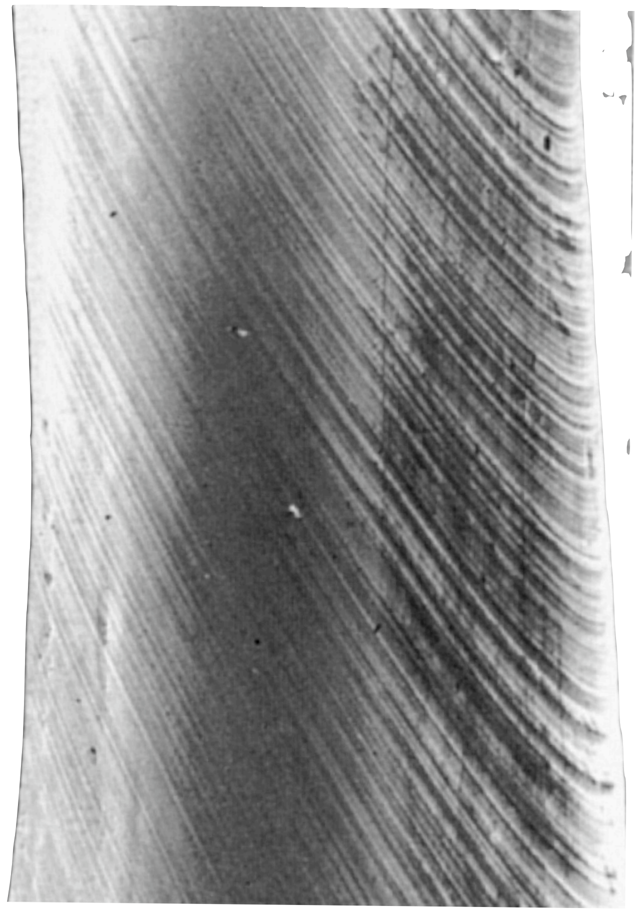
`crystallite.f90`

PART III

Monocrystal Plasticity Models

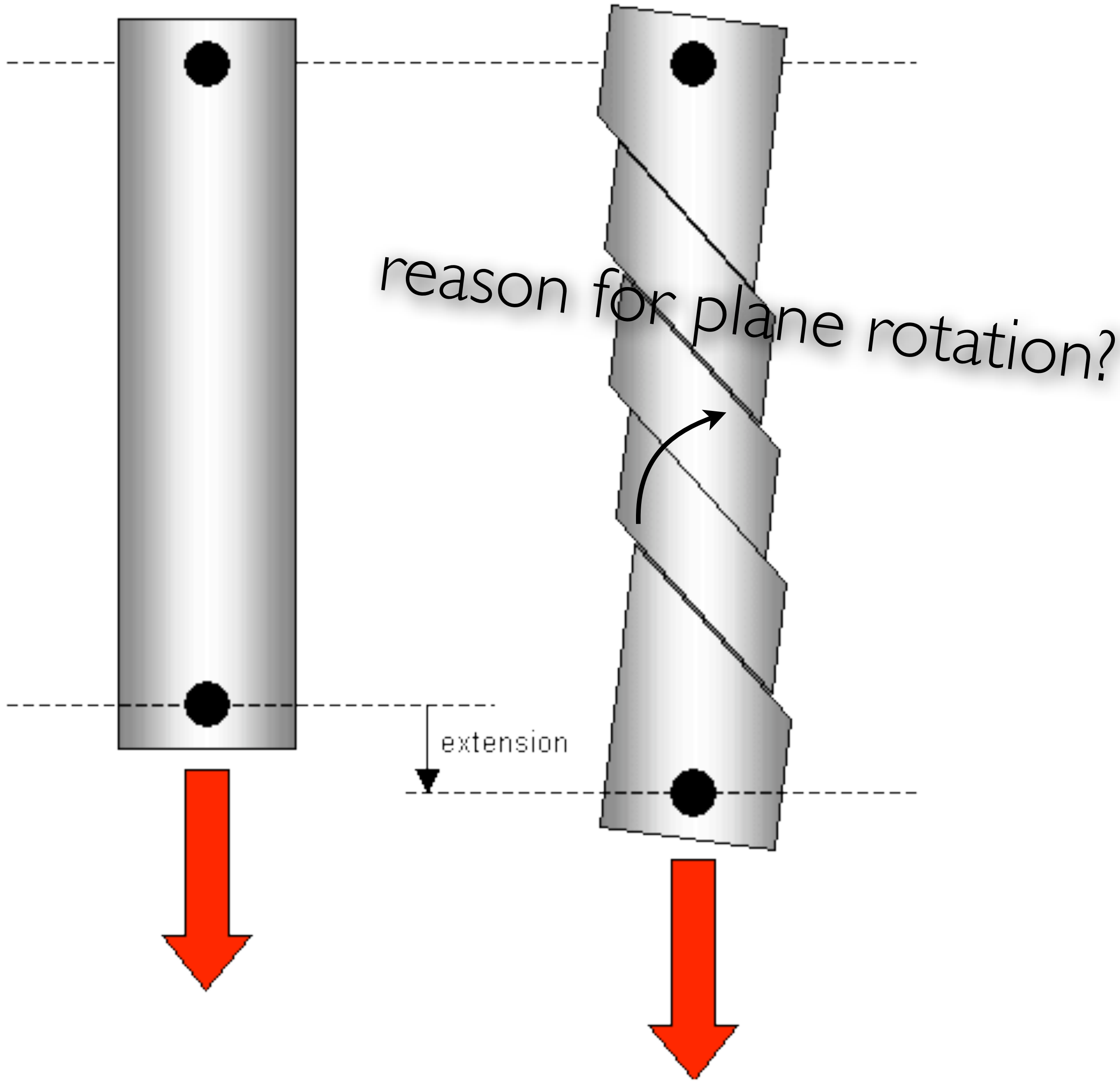
dislocation slip





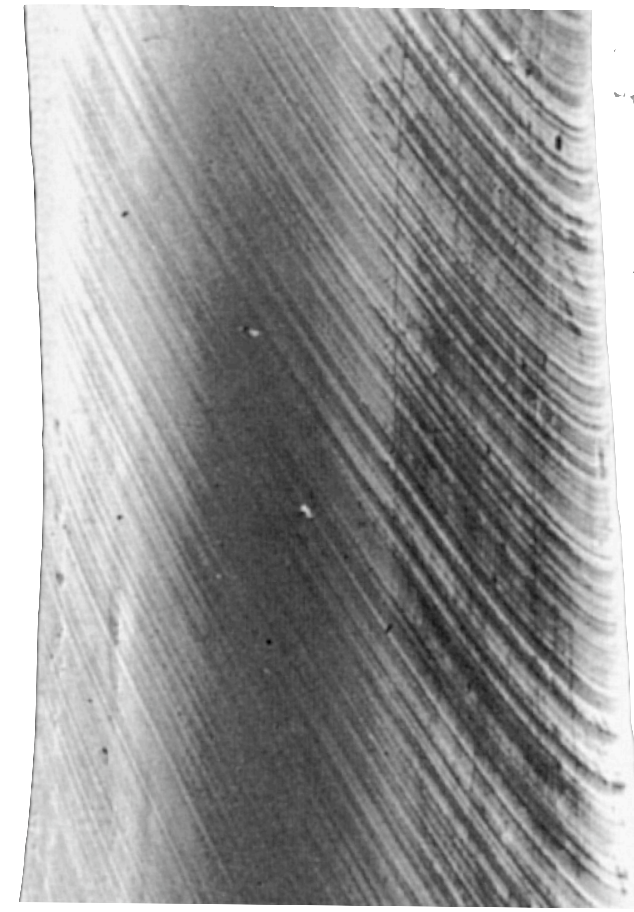
from www.msm.cam.ac.uk/doitpoms

dislocation slip



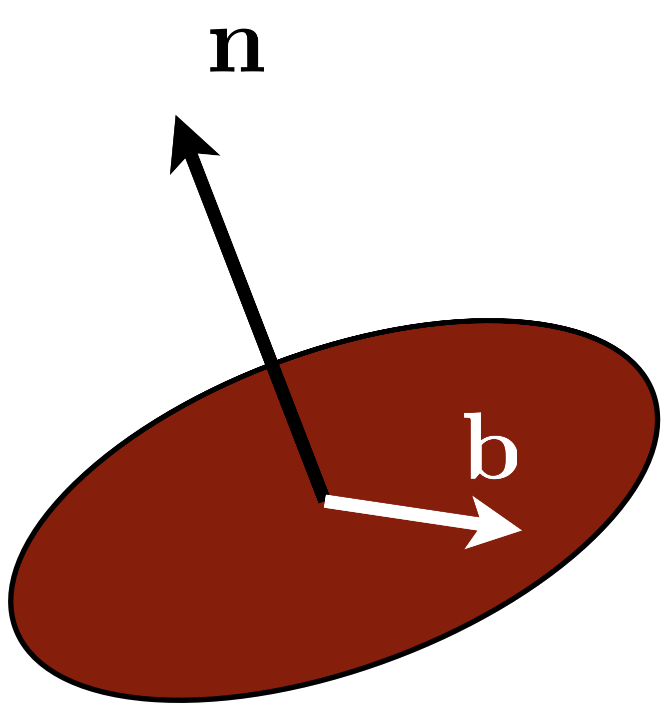
Unslipped single crystal fixed at top end.

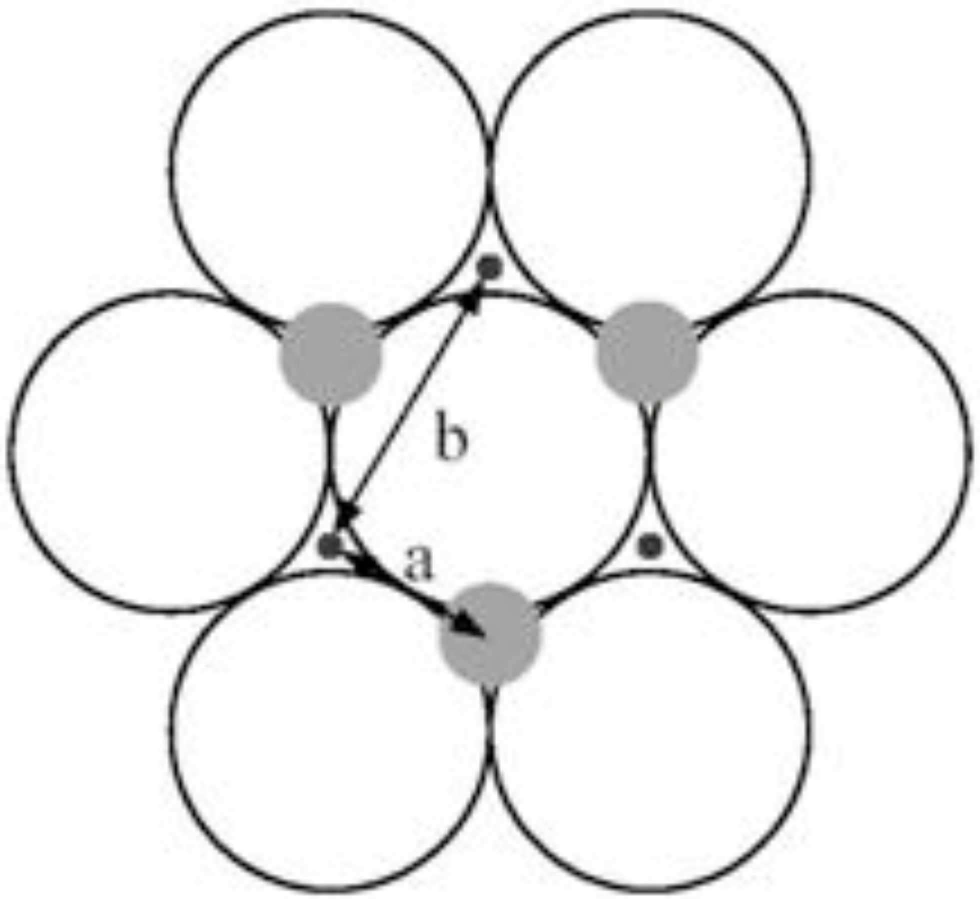
Single crystal after plastic deformation by tensile stress in the direction of the arrow. Slip occurs on distinct parallel planes.

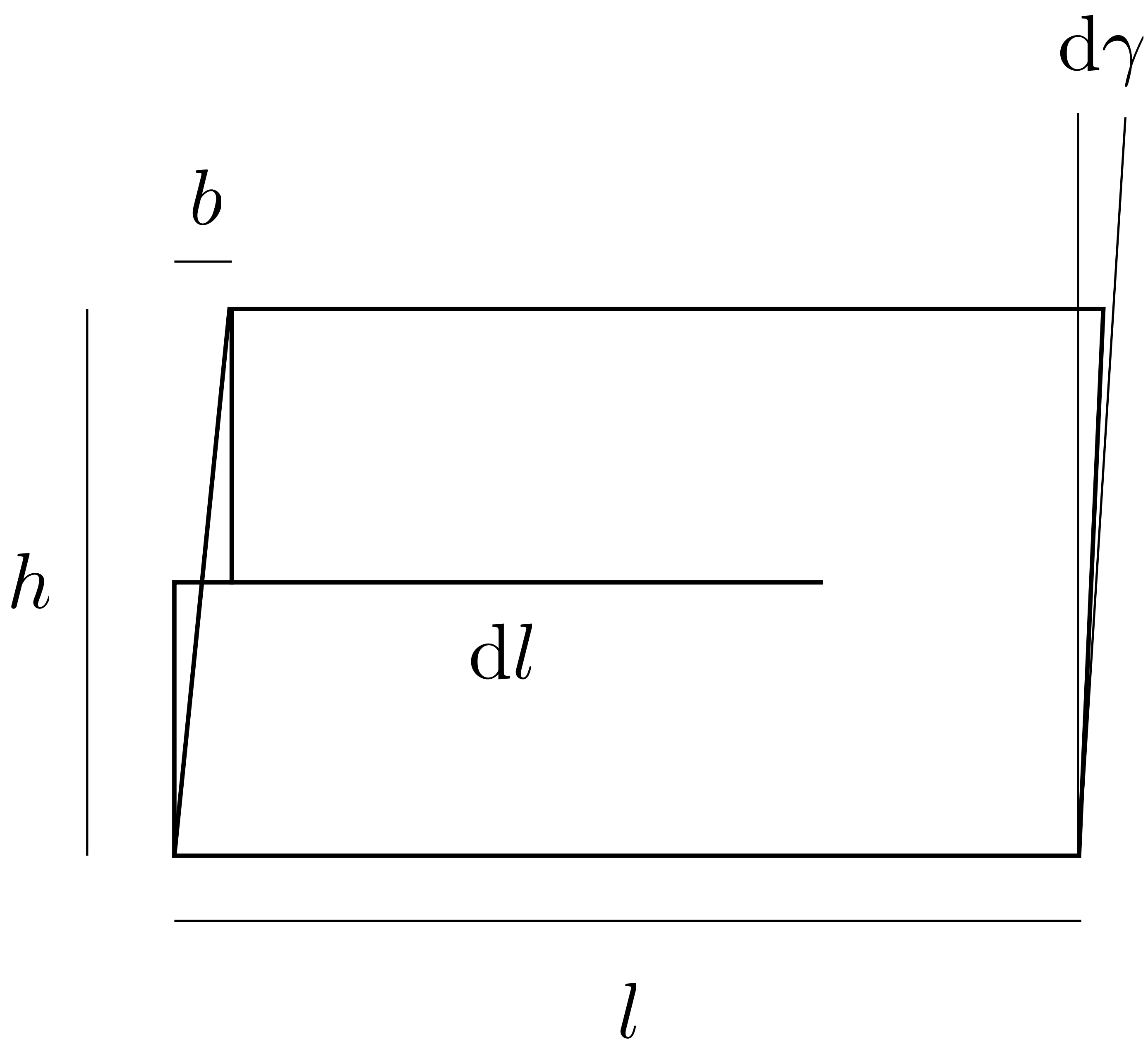


from www.msm.cam.ac.uk/doitpoms

slip systems in face centered cubic structure





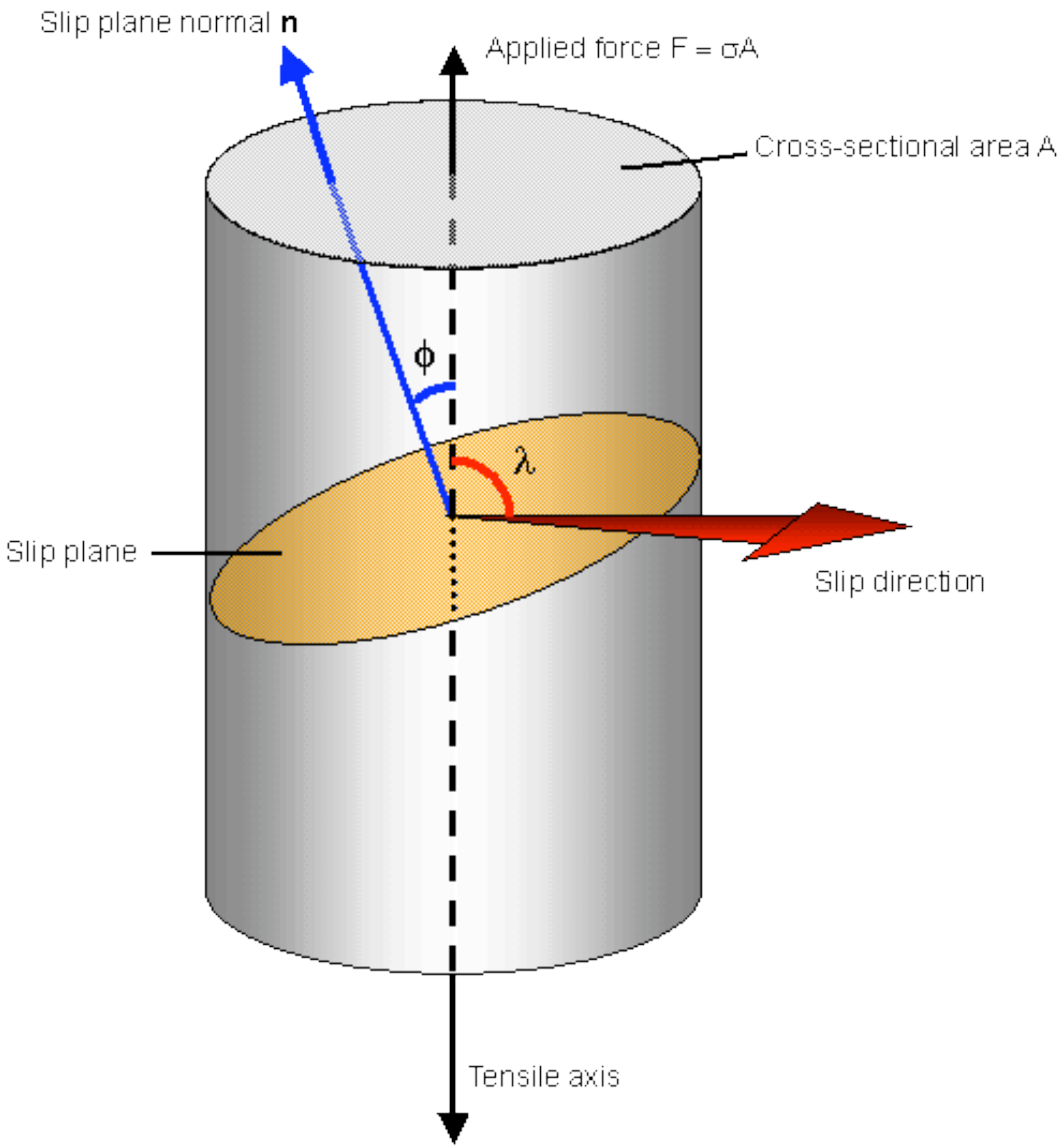


$$\mathrm{d}\gamma = \frac{b\,\mathrm{d}l}{h\,l} = \frac{b\,\mathrm{d}A}{V}$$

plastic velocity gradient

$$
\begin{aligned}
\mathbf{L}^{\mathrm{p}} &= (\mathbf{L}^{\mathrm{p}})_{ij} \\
&= \frac{\partial \dot{x}_i}{\partial x_j} \\
&= \sum_{\alpha} \dot{\gamma}^{(\alpha)} \mathbf{b}^{(\alpha)} \otimes \mathbf{n}^{(\alpha)}
\end{aligned}
$$

resolved shear stress

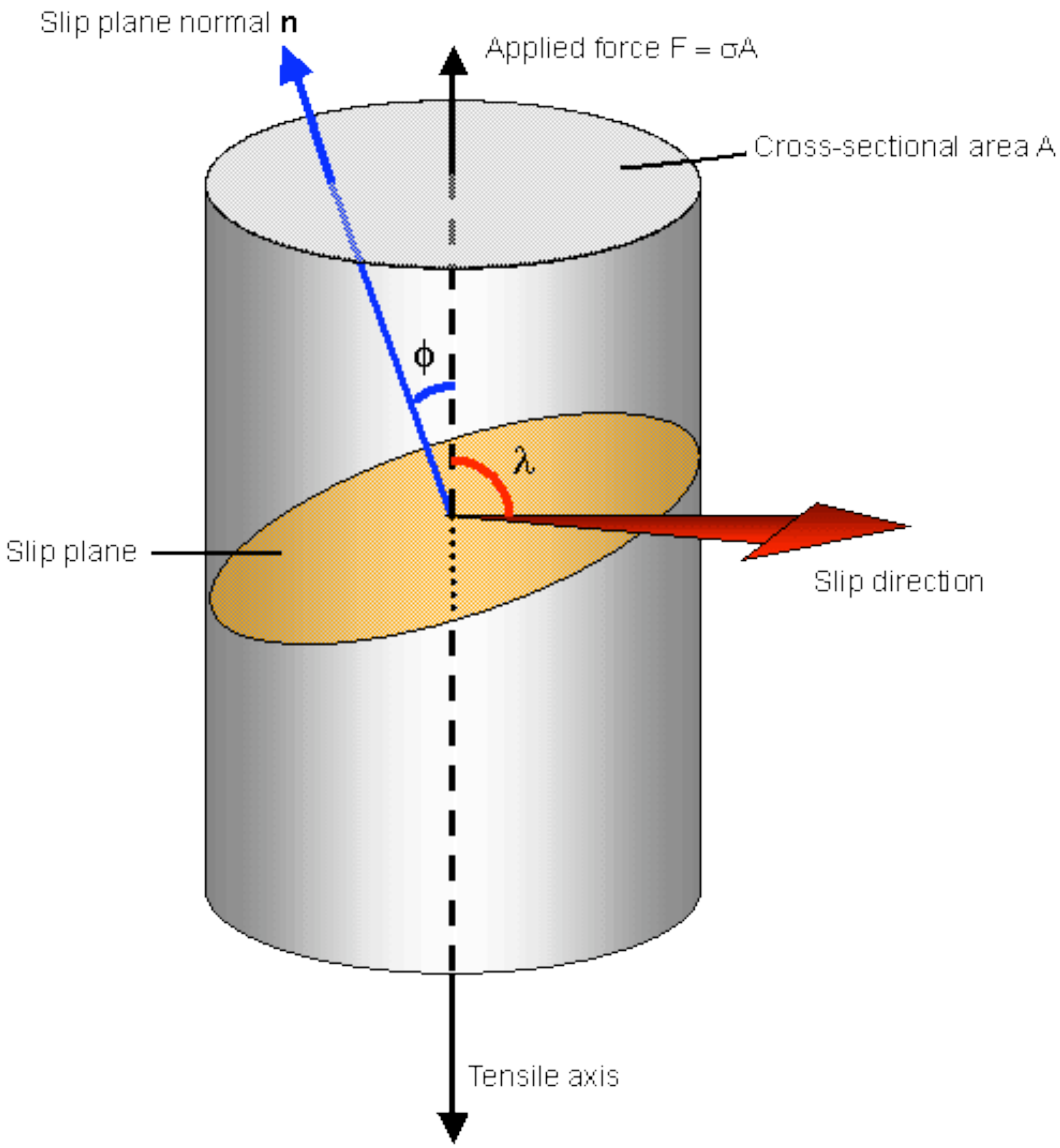


from www.msm.cam.ac.uk/doitpoms

uniaxial

$$\tau^{(\alpha)} = \sigma \cos\phi^{(\alpha)} \cos\lambda^{(\alpha)}$$

resolved shear stress



from www.msm.cam.ac.uk/doitpoms

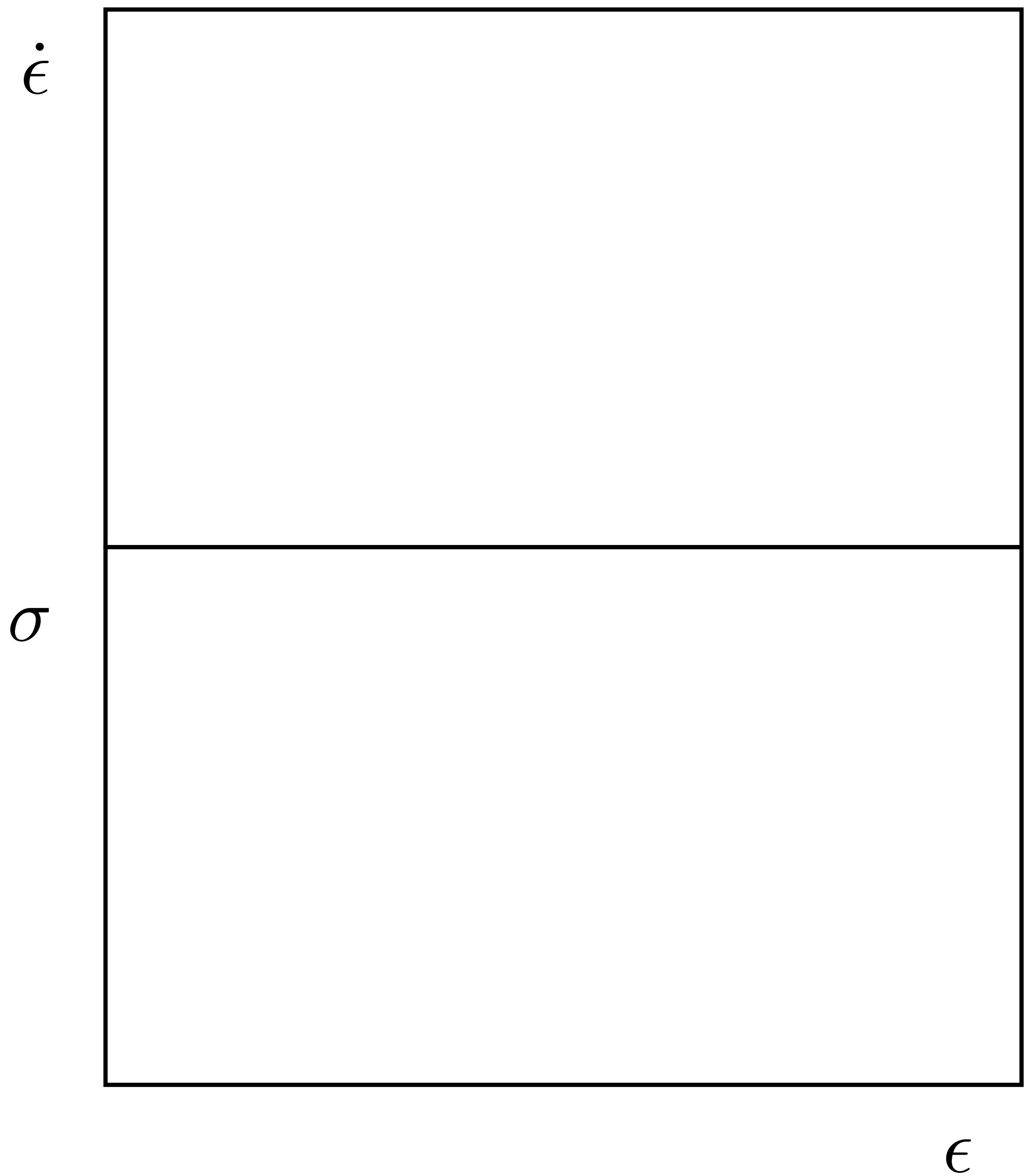
uniaxial

$$\tau^{(\alpha)} = \sigma \cos \phi^{(\alpha)} \cos \lambda^{(\alpha)}$$

general

$$\tau^{(\alpha)} = \mathbf{S} : (\mathbf{b}^{(\alpha)} \otimes \mathbf{n}^{(\alpha)})$$

room-temperature deformation resistance in fcc metals



- low strain-rate sensitivity
- largely monotonic decrease in strain hardening coefficient

phenomenological description

deformation kinetics

$$\dot{\gamma}^{(\alpha)} = \dot{\gamma}_0 \left| \frac{\tau^{(\alpha)}}{\tau_{\mathrm{c}}^{(\alpha)}} \right|^n \operatorname{sign} \tau^{(\alpha)}$$

microstructure evolution

$$\dot{\tau}_{\mathrm{c}}^{(\alpha)} = \sum_{\beta} q_{\alpha\beta}\, h_0 \left(1 - \frac{\tau_{\mathrm{c}}^{(\beta)}}{\tau_{\mathrm{s}}} \right)^a \dot{\gamma}^{(\beta)}$$

drawbacks

- independent of temperature
- independent of strain path
- independent of grain size

basis

$$\dot{\gamma} = \frac{b\,\dot{A}}{V} = b\,\frac{\ell}{V}\,\dot{w} = b\,\varrho_{\mathrm{m}}\,v$$

issues

- parameterization of microstructure
- velocity of dislocations
- evolution of microstructure

density on each system

$$\varrho^{(\alpha)} \quad \text{with } \alpha = 1, \ldots, 12$$

projected perpendicular density

$$\varrho_{\perp}^{(\alpha)} = \frac{1}{2}\sum_{\beta}\varrho^{(\beta)}\left[\left|\mathbf{n}^{(\alpha)}\cdot\left(\mathbf{n}^{(\beta)}\times\mathbf{b}^{(\beta)}\right)\right| + \left|\mathbf{n}^{(\alpha)}\cdot\mathbf{b}^{(\beta)}\right|\right]$$

projected parallel density

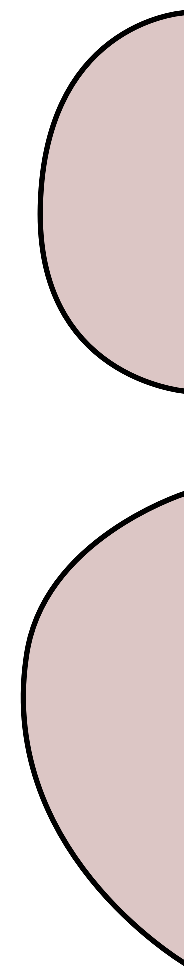
$$\varrho_{\parallel}^{(\alpha)} = \frac{1}{2} \sum_{\beta} \varrho^{(\beta)} \left[\|\mathbf{n}^{(\alpha)} \times (\mathbf{n}^{(\beta)} \times \mathbf{b}^{(\beta)})\| + \|\mathbf{n}^{(\alpha)} \times \mathbf{b}^{(\beta)}\| \right]$$

derived mobile density

$$
\begin{aligned}
\frac{\partial \tau^{(\alpha)}}{\partial \varrho_{\mathrm{m}}^{(\alpha)}} &= 0 \\
\Rightarrow \quad \varrho_{\mathrm{m}}^{(\alpha)} &\propto \sqrt{\varrho_{\|}^{(\alpha)} \varrho_{\perp}^{(\alpha)}}
\end{aligned}
$$

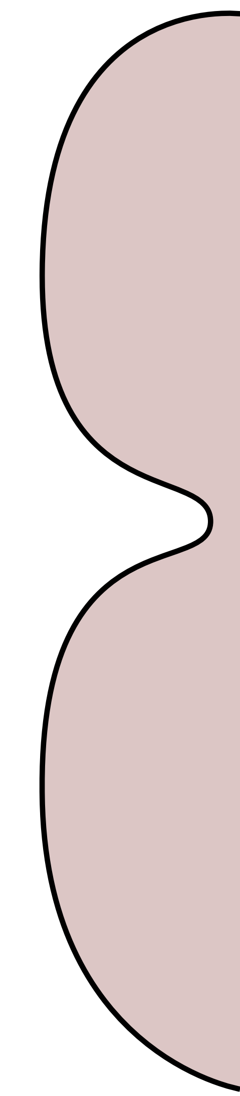
dislocation structure parameterization *(following Eisenlohr & Blum 2005)*

(specific) dipole density



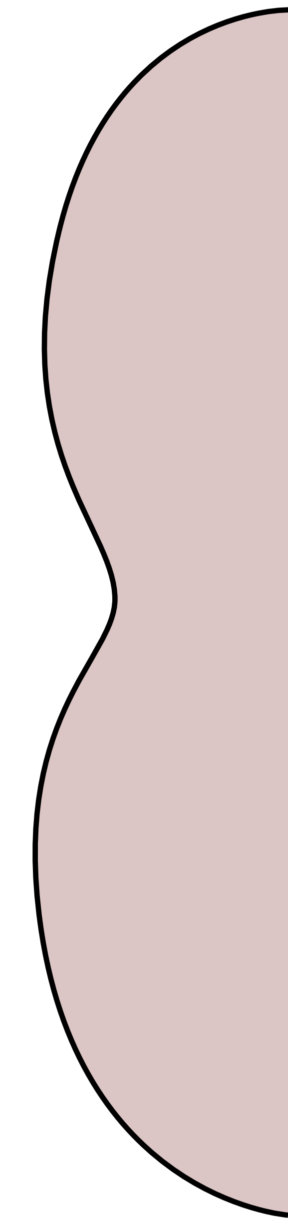
dislocation structure parameterization *(following Eisenlohr & Blum 2005)*

(specific) dipole density



dislocation structure parameterization *(following Eisenlohr & Blum 2005)*

(specific) dipole density



dislocation structure parameterization *(following Eisenlohr & Blum 2005)*

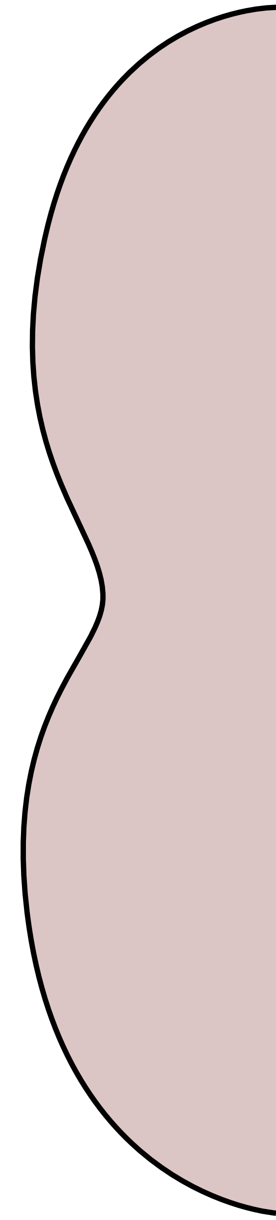
(specific) dipole density

$$\varrho_{\text{sgl}}^{(\alpha)} \quad \text{with } \alpha = 1, \ldots, 12$$

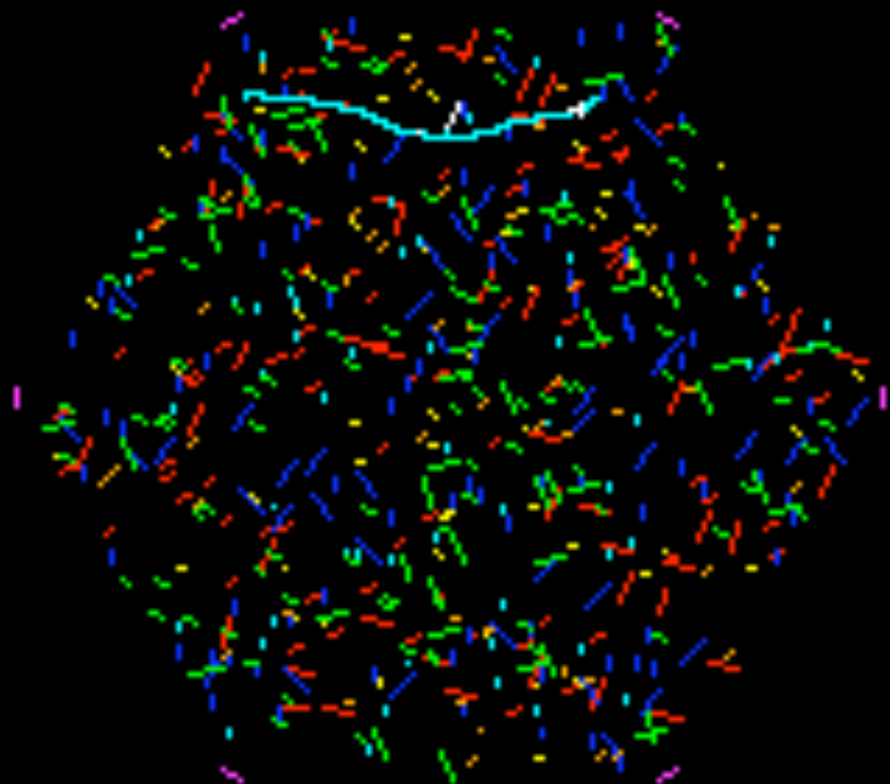
$$\varrho_{\text{dip}}^{(\alpha)} \quad \text{with } \alpha = 1, \ldots, 12$$

$$\chi^{(\alpha)} \equiv \frac{\partial \varrho_{\text{dip}}^{(\alpha)}}{\partial h} \quad \text{with } \alpha = 1, \ldots, 12$$

$$h_{\text{spon}} < h < \hat{h}^{(\alpha)} \equiv \frac{1}{8\pi(1-\nu)} \frac{G\, b}{\tau^{(\alpha)}}$$

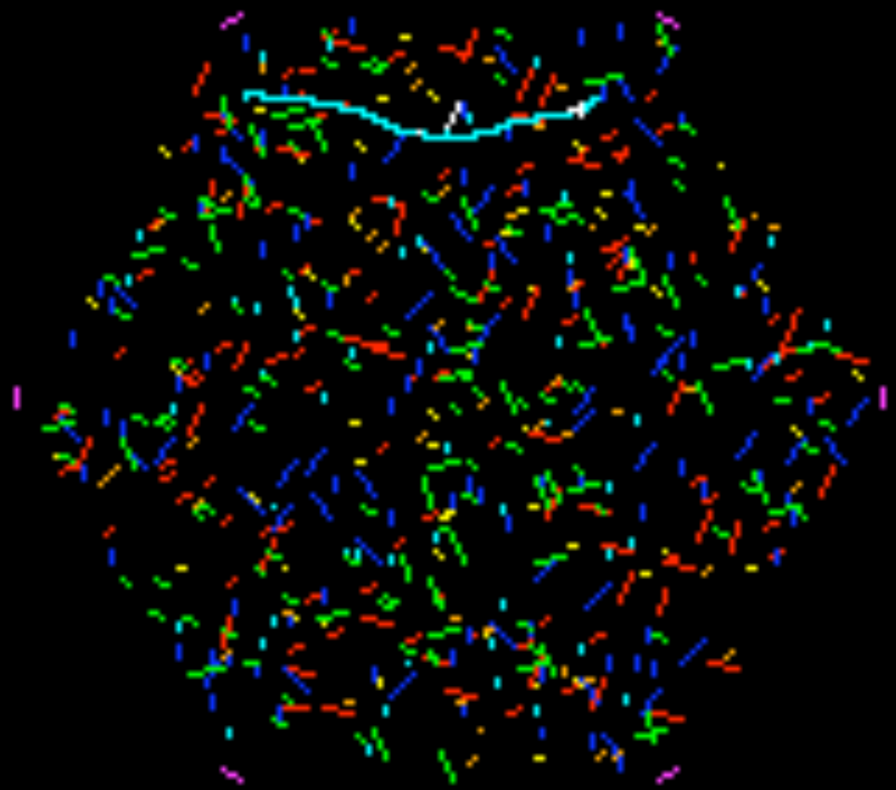


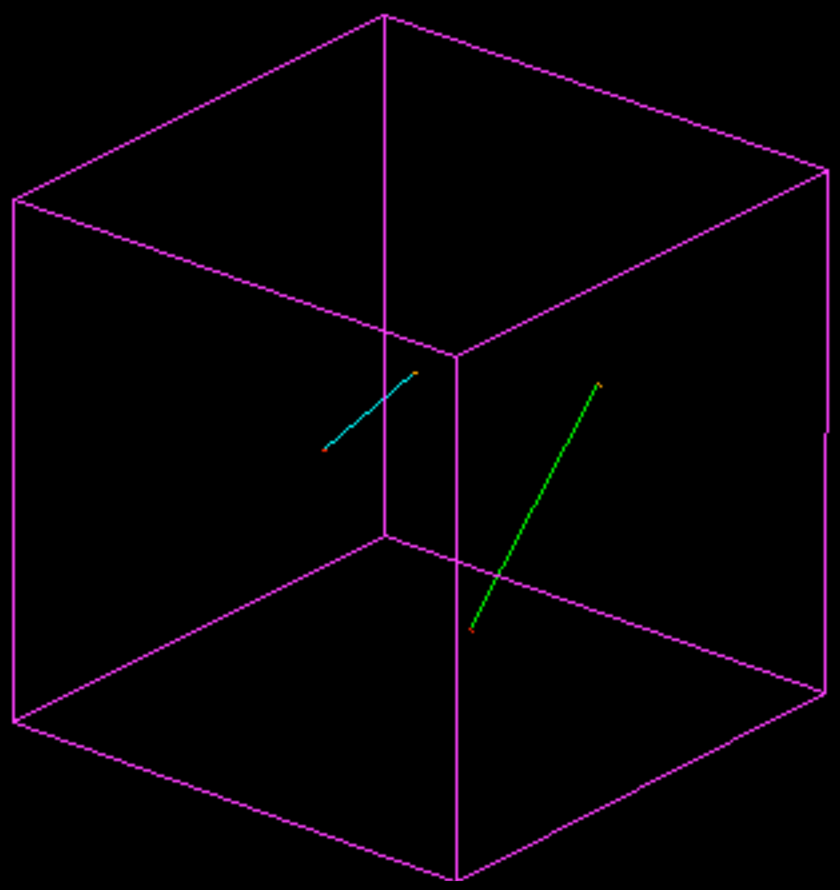
dislocation–forest interaction



from zig.onera.fr

dislocation–forest interaction

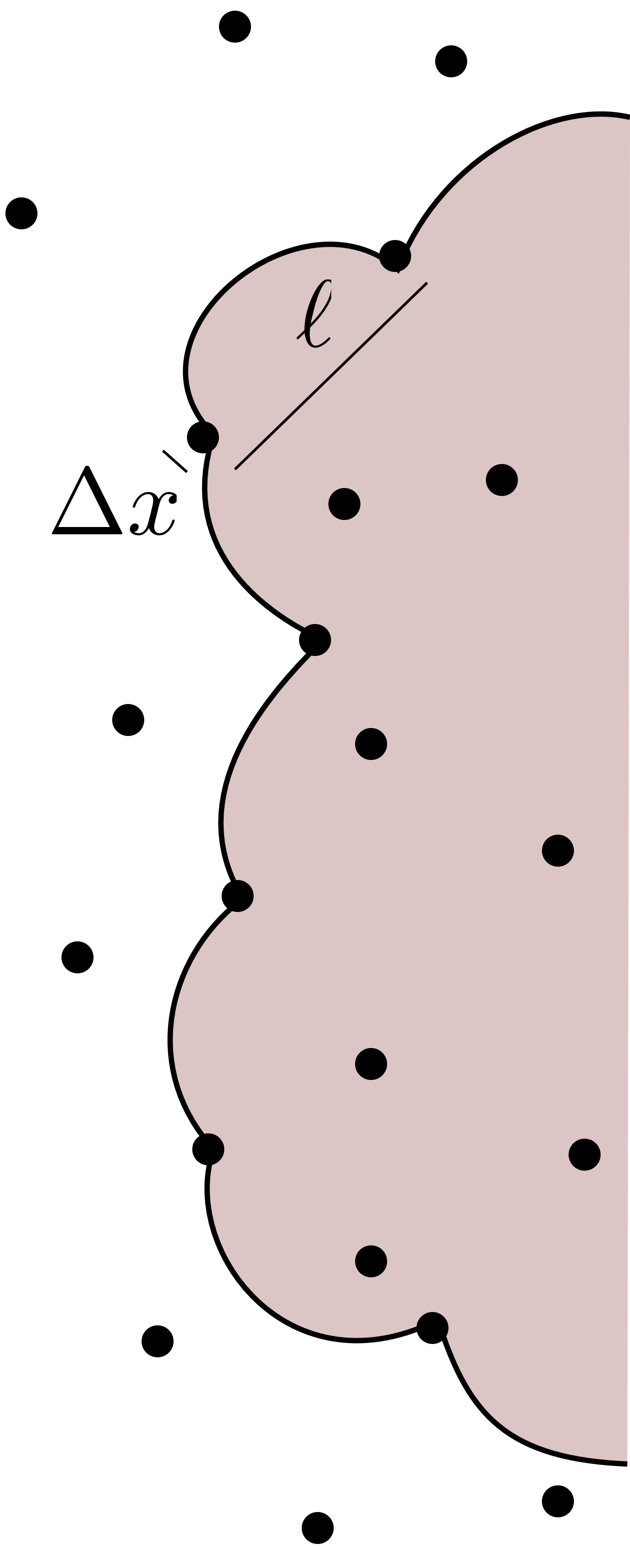




from zig.onera.fr

thermally activated forest cutting

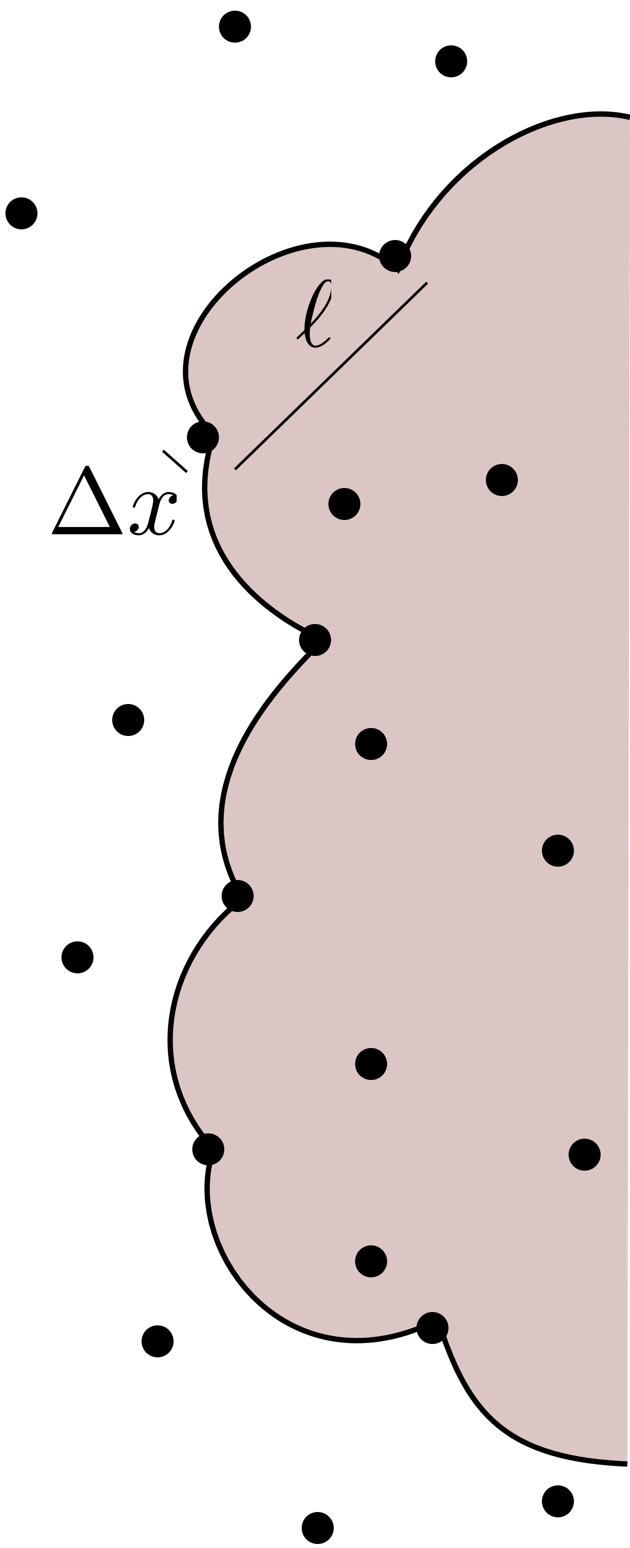
$$v^{(\alpha)} = \lambda^{(\alpha)} \nu_{\mathrm{attack}} \, \mathrm{sign}\left(\tau^{(\alpha)}\right) \times \exp\left(-\frac{Q_{\mathrm{slip}}}{k_{\mathrm{B}}T}\right) \sinh\left(\frac{\tau_{\mathrm{eff}}^{(\alpha)} V^{(\alpha)}}{k_{\mathrm{B}}T}\right)$$



thermally activated forest cutting

$$v^{(\alpha)} = \lambda^{(\alpha)} \nu_{\text{attack}} \operatorname{sign}\left(\tau^{(\alpha)}\right) \times \exp\left(-\frac{Q_{\text{slip}}}{k_{\text{B}} T}\right) \sinh\left(\frac{\tau_{\text{eff}}^{(\alpha)} V^{(\alpha)}}{k_{\text{B}} T}\right)$$

$$V^{(\alpha)} = b\,\ell\,\Delta x$$

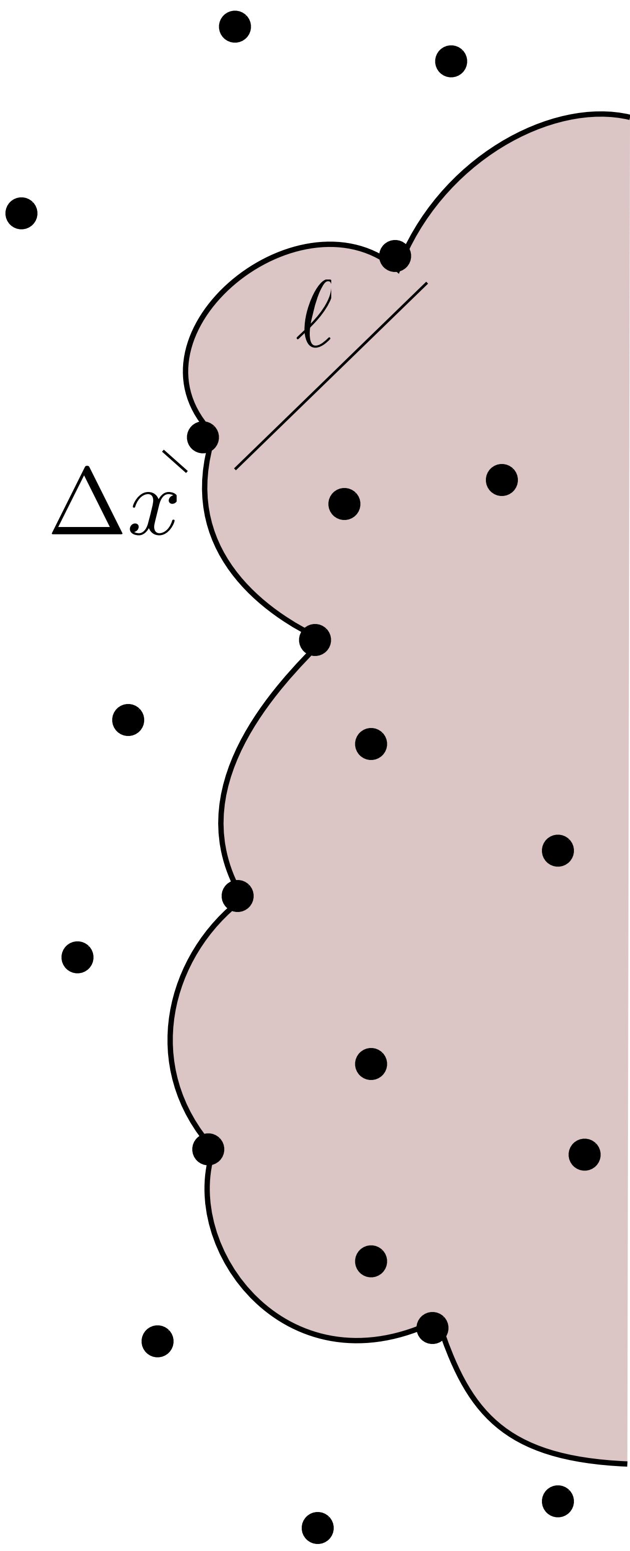


thermally activated forest cutting

$$v^{(\alpha)} = \lambda^{(\alpha)} \nu_{\text{attack}} \,\text{sign}\left(\tau^{(\alpha)}\right) \times \exp\left(-\frac{Q_{\text{slip}}}{k_{\text{B}} T}\right) \sinh\left(\frac{\tau_{\text{eff}}^{(\alpha)} V^{(\alpha)}}{k_{\text{B}} T}\right)$$

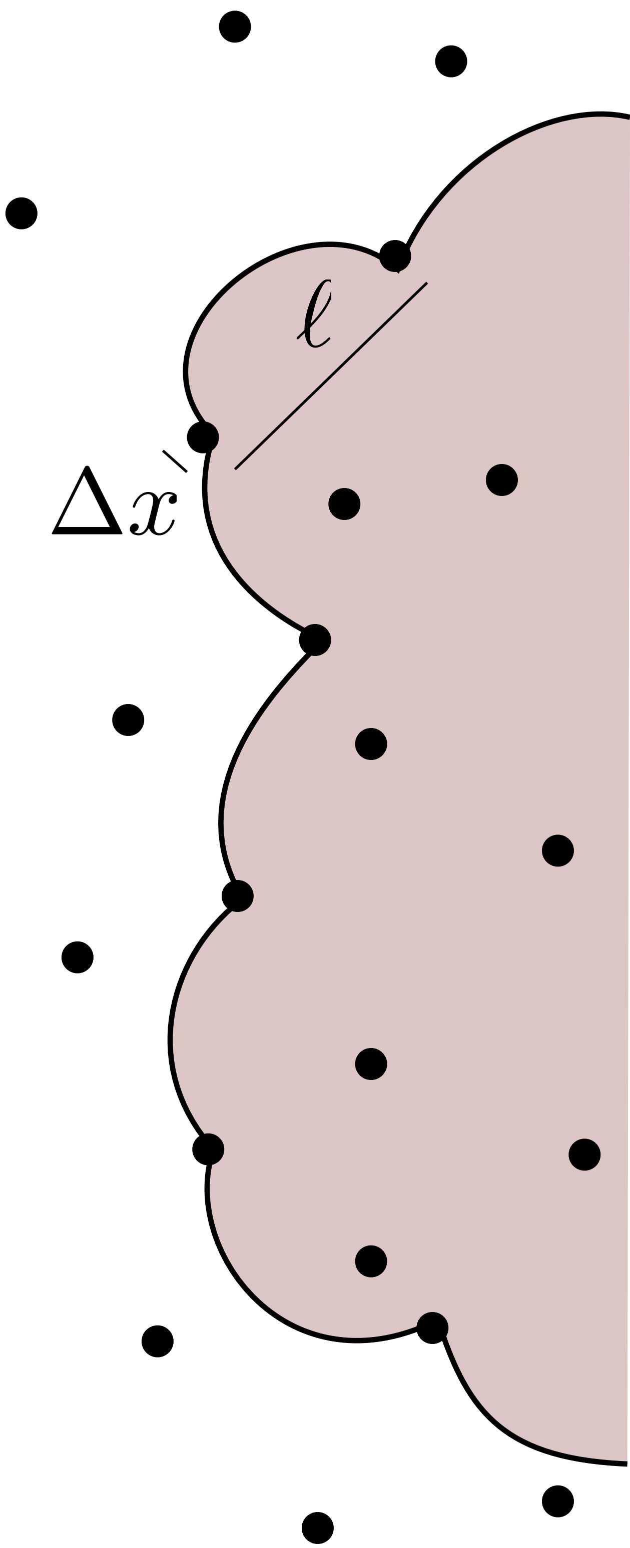
$$V^{(\alpha)} = b\,\ell\,\Delta x$$

$$\ell \propto \left(\varrho_{\perp}^{(\alpha)}\right)^{-0.5}$$



thermally activated forest cutting

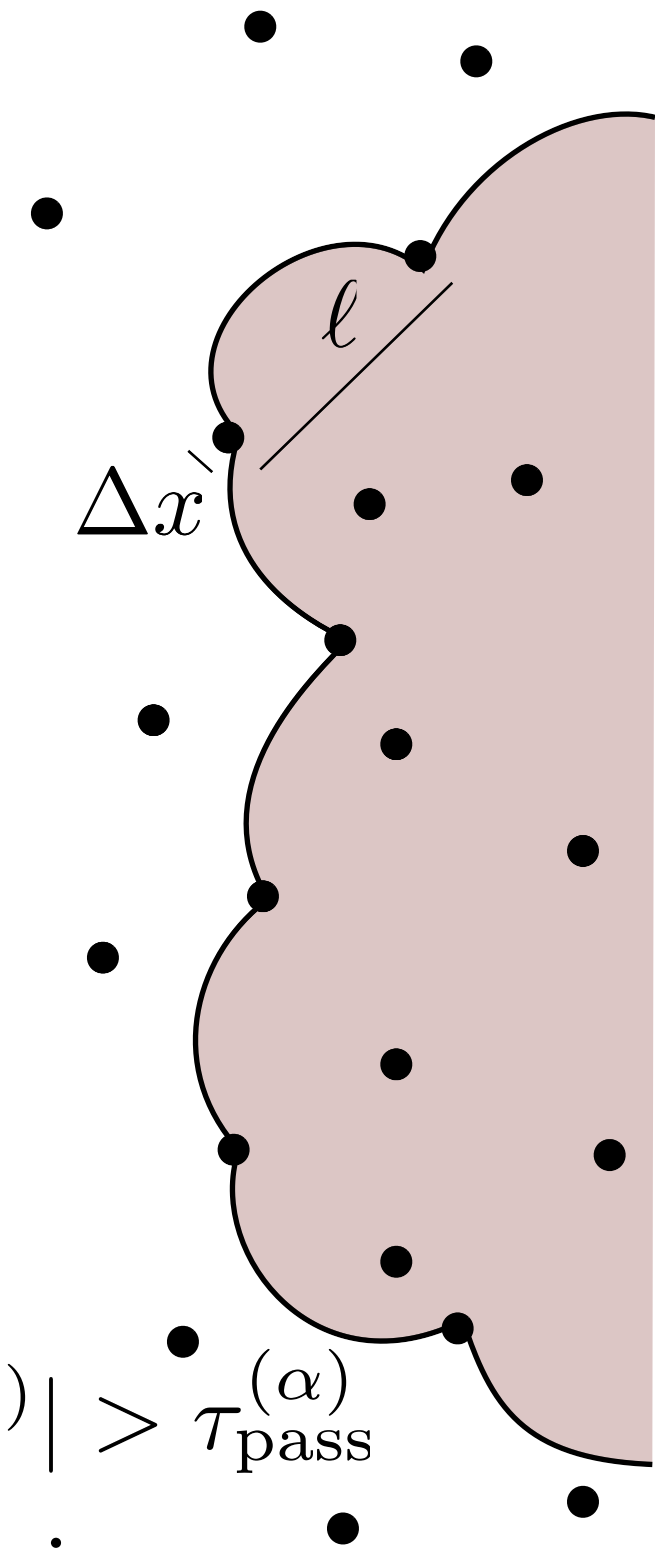
$$v^{(\alpha)} = \lambda^{(\alpha)} \nu_{\text{attack}} \, \text{sign}\left(\tau^{(\alpha)}\right) \times \exp\left(-\frac{Q_{\text{slip}}}{k_{\text{B}} T}\right) \sinh\left(\frac{\tau_{\text{eff}}^{(\alpha)} V^{(\alpha)}}{k_{\text{B}} T}\right)$$



thermally activated forest cutting

$$v^{(\alpha)} = \lambda^{(\alpha)} \nu_{\text{attack}} \, \text{sign}\left(\tau^{(\alpha)}\right) \times \exp\left(-\frac{Q_{\text{slip}}}{k_{\text{B}} T}\right) \sinh\left(\frac{\tau_{\text{eff}}^{(\alpha)} V^{(\alpha)}}{k_{\text{B}} T}\right)$$

$$\tau_{\text{eff}}^{(\alpha)} = |\tau^{(\alpha)}| - \tau_{\text{pass}}^{(\alpha)} = \begin{cases} |\tau^{(\alpha)}| - c_3 \, G \, b \sqrt{\varrho_{\parallel}^{(\alpha)} + \varrho_{\text{m}}^{(\alpha)}} & \text{if } |\tau^{(\alpha)}| > \tau_{\text{pass}}^{(\alpha)} \\ 0 & \text{otherwise} \end{cases}$$



dislocation structure evolution

$$\dot{\chi} = \frac{2\dot{\gamma}}{bN_\mathrm{g}} \left[\overbrace{\varrho_\mathrm{sgl}}^{\text{dipole formation}} + \overbrace{\int_h^{\hat{h}} \chi \, \mathrm{d}h' - \chi h}^{\text{exchange of constituents}} - \overbrace{\chi h \left(\frac{1}{1 - 0.9h/\hat{h}} - 1 \right)}^{\text{dipole decompostion}} \right] + \underbrace{\frac{2v_\mathrm{c}\chi}{h} \left(\frac{\partial(\ln \chi)}{\partial(\ln h)} - 1 \right)}_{\text{climb of constituents}}$$

$$\dot{\varrho}_\mathrm{sgl} = \frac{2\dot{\gamma}}{b\,N_\mathrm{g}} \left(\overbrace{\frac{N_\mathrm{g}}{2\Lambda}}^{\text{dislocation generation}} - \overbrace{\hat{h}\varrho_\mathrm{sgl}}^{\text{dipole formation}} + \overbrace{\int_{h_\mathrm{spon}}^{\hat{h}} \chi h \left(\frac{1}{1 - 0.9h/\hat{h}} - 1 \right) \mathrm{d}h}^{\text{dipole decomposition}} \right)$$

check out the source code...

`constitutive.f90`